Basics of Computing – Chapter 1.1-1.3
Data Manipulation and Storage Media

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Boolean Symbol System

Circuits

Data Storage

0's and 1's

Binary **number system** vs. Boolean **symbol system**

Binary:

- Operations: $+$, $\times$, $/$, $-$
- $\{0, 1, 10, 11, 100, 101, \ldots\}$

Boolean:

- Operations: AND, OR, XOR, NOT, . . .
- $\{0, 1\}$ or $\{\text{true}, \text{false}\}$ or $\{\text{on}, \text{off}\}$

Boolean Logic
Binary Nature

Boolean logic is binary in nature, thus there are two values for:

- Any given condition
- Any answer based on the condition

For example, is the weather pleasant?

- Three conditions: Wind, warmth, and rain.
- These values can be set to either true (1), or false (0).

Today: warm = false, wind = false, rain = true.
Weather pleasant? **False**

Allowing degrees of warmth, wind, and rain would require levels of each condition.

Boolean Logic

Gate

The circuitry to carry out boolean logic are called **logical gates**.

- Designed to accept a small number of conditions
- Conditions expressed as `true` or `false`

Consider logic gates with only two inputs. For a given gate, there will be particular response if:

1. Both inputs are `false`,
2. Only the first input is `false`,
3. Only the first input is `true`, and
4. Both inputs are `true`.

To show the output of a gate for each pair of inputs, we build **truth tables**.

Truth Tables

Truth tables compute functional values of logical expressions w.r.t. all possible combinations their logical variables may take.

Column headings on a truth table show:

1. Inputs (variables)
2. Expression result, based on variables and operators

Row headings show:

1. All possible T/F assignment to variables
2. Result of T/F assignment to Expression

Inputs:		Outputs:	
P	Q	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Boolean Logic

Operations and Circuitry

- Input: 2 bits
- Number of operations for 2-bit input operations: $2^4 = 16$
- Given the following setup for a truth table:

P	Q	P op Q
0	0	X_1
0	1	X_2
1	0	X_3
1	1	X_4

where result $X_i \in \{0, 1\}$.

16 combinations possible using the given setup:

$X_1X_2X_3X_4$	Description	Gate Name?
0000	Inputs ignored	*False*
0001	P and Q true	**AND**
0010	Only Q true	Q AND NOT P
0011	Q true	*Q*
0100	Only P true	P AND NOT Q
0101	P true	*P*
0110	Only P or Q true	**XOR**
0111	P or Q true	**OR**
1000	P and Q false	NOR
1001	P = Q	NXOR
1010	P false	**NOT P**
1011	Not only P true	P implies Q
1100	Q false	**NOT Q**
1101	Not only Q true	Q implies P
1110	P or Q false	NAND
1111	Inputs ignored	*True*

Boolean Logic
Operations

Out of the 16 possible gates, we will study only four:

- Three basic gates:
  1. NOT
  2. AND
  3. OR
- Combinations of these gates construct our other common gate:
  1. XOR

Boolean Operations
NOT – (NOT P, NOT Q)

The simplest boolean operation is **NOT**.

- Takes 1 input and inverts it.

NOT $0 = 1$

NOT $1 = 0$

P	Q	NOT P	NOT Q
0	0	1	1
0	1	1	0
1	0	0	1
1	1	0	0

Boolean Gates

NOT[1]

NOT

Inputs	Output
0	1
1	0

[1]From Pearson Education, Inc.

Boolean Operations
AND

Boolean AND works similar to binary multiplication.

- Takes 2 inputs, P and Q.

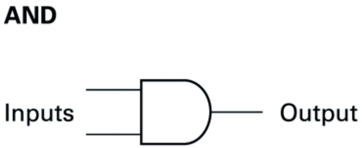
$$\begin{array}{r} 0 \\ \text{AND } 0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 0 \\ \text{AND } 1 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 1 \\ \text{AND } 0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1 \\ \text{AND } 1 \\ \hline 1 \end{array}$$

P	Q	P AND Q
0	0	0
0	1	0
1	0	0
1	1	1

Boolean Gates

AND[2]



Inputs	Output
0 0	0
0 1	0
1 0	0
1 1	1

[2] From Pearson Education, Inc.

Boolean Operations
OR

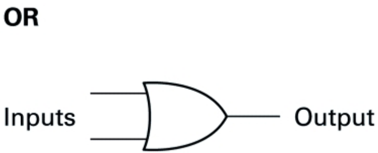
Boolean OR works similar to binary addition.

- Takes 2 inputs, P and Q.

0 OR 0 = 0	0 OR 1 = 1
1 OR 0 = 1	1 OR 1 = 1

P	Q	P OR Q
0	0	0
0	1	1
1	0	1
1	1	1

Boolean Gates
OR[3]



Inputs	Output
0 0	0
0 1	1
1 0	1
1 1	1

[3] From Pearson Education, Inc.

Boolean Operations
XOR

$$\text{Boolean XOR} = \begin{cases} |\ \text{input } 1's \text{ even}\ | & \text{output } 0 \\ |\ \text{input } 1's \text{ odd}\ | & \text{output } 1 \end{cases}$$

- Takes 2 inputs, P and Q.

```
    0
XOR 0
-----
    0
```

```
    0
XOR 1
-----
    1
```

```
    1
XOR 0
-----
    1
```

```
    1
XOR 1
-----
    0
```

P	Q	P XOR Q
0	0	0
0	1	1
1	0	1
1	1	0

Boolean Gates

XOR[4]

XOR

Inputs	Output
0 0	0
0 1	1
1 0	1
1 1	0

[4] From Pearson Education, Inc.

Boolean Symbol System

Circuits

Data Storage

Creating a Circuit

P	Q	P XOR Q
0	0	0
0	1	1
1	0	1
1	1	0

As it was previously mentioned, the XOR gate can be produced by the three basic gates

- AND
- OR
- NOT

Creating a Circuit
Steps

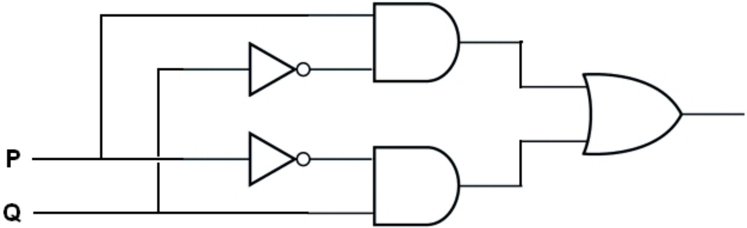
P	Q	P XOR Q
0	0	0
0	1	1
1	0	1
1	1	0

1. Highlight the rows in which a 1 shows up in the output.
2. For each row, use AND and NOT gates to make the inputs equal 1.

   r_{01} (NOT P AND Q)
   r_{10} (P AND NOT Q)
3. If there is more than 1 row highlighted, connect gates with OR
   - (NOT P AND Q) OR (P AND NOT Q)
4. Draw circuit (on board)

Creating a Circuit

XOR using AND's and NOT's



Boolean Addition

Two-bit Adder

Addition can be performed using the general equation:

$$\begin{array}{r} P \\ +Q \\ \hline CS \end{array}$$

P and Q are the inputs, C is the **carry** bit, and S is the **sum** bit. To add two bits, we need to match all four cases of addition:

$$\begin{array}{r} 0 \\ +0 \\ \hline 00 \end{array} \qquad \begin{array}{r} 0 \\ +1 \\ \hline 01 \end{array} \qquad \begin{array}{r} 1 \\ +0 \\ \hline 01 \end{array} \qquad \begin{array}{r} 1 \\ +1 \\ \hline 10 \end{array}$$

The sum in all 4 cases is two bits, therefore, two outputs. This implies that there needs to be at least two logic gates.

Boolean Addition

Circuit Creation

To create a circuit with more than one output, create a circuit for each output.

P	Q	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Boolean Addition

Two-bit Adder

Highlight the rows with 1's in the output. *Note that each output (S and C) is treated independently of the other!*

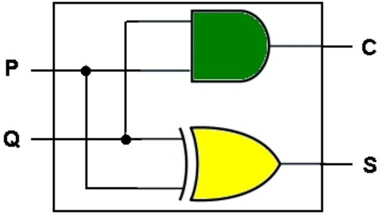
P	Q	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

P	Q	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Boolean Addition

Two-bit Adder

P	Q	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

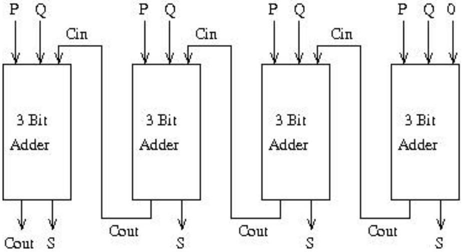


Boolean Addition

Three-bit Adder

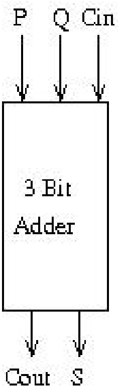
The two-bit adder is restricted to adding bit-strings of length 1 only.

Multi-column addition is preferable. For this, we need an adder that can propagate carry bits:



Boolean Addition

Three-bit Adder



The Logic table for a 3-bit Adder:

P	Q	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Boolean Addition
Three-bit Adder

3-bit Adder:

P	Q	Cin	Cout	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

C_{out}:

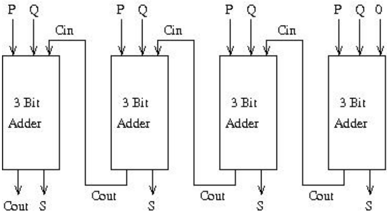
- Occurs when either P & Q, P & C_{in}, or Q & C_{in} are 1.
- Bullet point above: 3 &'s, 1 or!

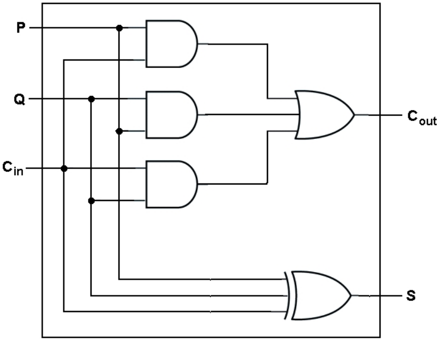
S:

- True when an odd number of inputs are 1.
- Suggests we need only to compute the sum, as in the two-bit adder
  - 3-bit XOR: (P XOR Q) XOR C_{in}

Boolean Addition

Three-bit Adder





Flip-Flops

In previous digital computations, the inputs were given to a gate, and stage-by-stage, the output of each gate is found by a lookup in it's logic table, until the end of the computation is reached, and the output is determined.

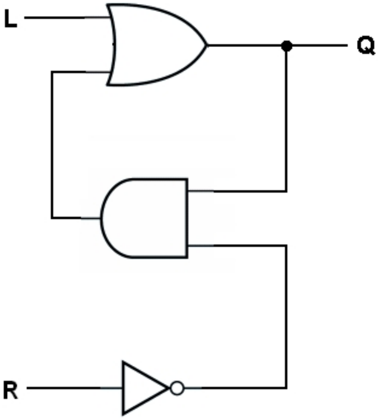
More difficult (though important) computations require *feedback* from outputs into earlier inputs. One example of such a gate is the **flip-flop**.

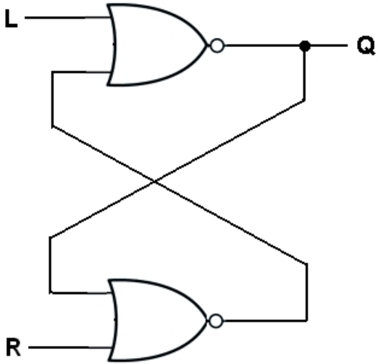
Definition

A flip-flop is a circuit that produces an output value of 0 or 1, which remains constant until a temporary pulse from another circuit causes it to shift to the other value.

Flip-Flops

Like a Light Switch





Flip-Flops

Why?

Once set, flip-flops will maintain it's current state (output value). Other circuits can change the state of the flip-flop by sending a **pulse** (1-bit) to the inputs of the flip-flop.

- Maintenance of the output state $\longleftarrow$ storing a bit within a computer.
- To store information, many flip-flops can be constructed on a **computer chip**.

Boolean Symbol System

Circuits

Data Storage

Storage Devices in Computer Systems

Issues

Storing "stuff" brings up many issues, depending on what it is:

- How much storage do we need?
- How do we get our stuff?
- How fast can we get our stuff?
- How much are we willing to spend on our storage materials?
- How do we organize our stored stuff?
- How long will the items last?

We can answer these questions for many things:

- Food,
- papers (like old homework),
- etc.

Storage Devices in Computer Systems

Bits and Bytes

- How much storage do we need? **Size**
- How do we get our stuff? **Access Method**
- How fast can we get our stuff? **Access Speed**
- How much are we willing to spend on our storage materials? **Cost**
- How do we organize our stored stuff? **Organization and Structure**
- How long will the items last? **Life-time of stored elements**.

It is important to understand the impact of each of these questions when working with computers.

Storage Devices in Computer Systems

Phone Book

 [a]

- We want to find the phone number of someone fast
- Assume phone book is not in alphabetical order
  - Sequentially search entry-by-entry.

Very slow.

Sequential.

[a] http://www.lynnlake.ca

Storage Devices in Computer Systems

Paper Clipped Numbers

[a]

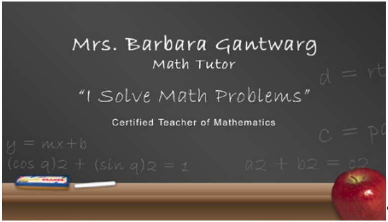
- We keep the business cards of numbers we use frequently in our wallet.
  - Look at the cards on a regular basis.
  - Faster and easier than the phonebook.
  - Wallet is small.
  - Still flip through the cards.

Faster than phone book, but still slow. Sequential or Random?

[a] http://www.sfc-webdesign.com

Storage Devices in Computer Systems

Number that is Found, taken from paper clip.



[a] http://www.toadwebsites.com

- Making calls, take a couple of cards out.
  - Find correct number on business card (Fax, office, home, ...)
  - Business card is very small.
  - Easier to find number.
  - Limitations?

Fast.

Random

Storage Devices in Computer Systems

Written Number

867-5309

- Found the number, write it down
  - Very fast.
  - Cannot write more than one number.
  - After phone call, what happens to number?

Very Fast.
Random.

Comparisons of Storage Options

	Speed	Size	Volatility	Access Method
Phone Book	Very Slow	Very Large	Very Low	Sequential
Paper Clip	Slow	Large	Low	Random
Biz. Card	Fast	Small	High	Random
Whiteboard	Very Fast	Very Small	Very high	Random

Storage Devices in Computer Systems
From Slowest to Fastest

- Mass Storage: Hard Drive, CD, DVD, Tape backup
- Main Memory
- Cache
- Registers

Storage Devices in Computer Systems
Main Memory

Memory is stored in units called cells.

- Each cell is 1 byte.
- each cell has a “name”, or address.
  - Similar to house address
- Main Memory is usually called *Random Access Memory* (RAM)
  - We can access any cell directly, without scanning other cells.

Cell 0	01011011
Cell 1	01001111
Cell 2	01100111
Cell 3	00000010
Cell 4	01101011
Cell 5	10001101
Cell 6	00110010
Cell 7	10111011
⋮	⋮
Cell n	11011011

Storage Devices in Computer Systems
Main Memory

- There are two operations in memory:
  - Read
  - Write
- Read(Cell 5) $\longrightarrow$ 10001101
- Write(00000000, Cell 5) replaces the contents of Cell 5 with 00000000.

Cell 0	01011011
Cell 1	01001111
Cell 2	01100111
Cell 3	00000010
Cell 4	01101011
Cell 5	10001101
Cell 6	00110010
Cell 7	10111011
⋮	⋮
Cell n	11011011

Storage Devices in Computer Systems
Main Memory – Pro's

Main memory plays a vital role in any computer

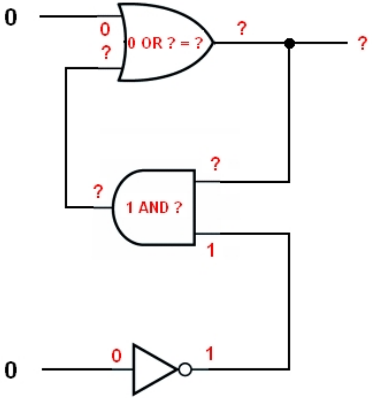
- All read and write operations go through the Main memory
- Main memory is faster than the Hard drive

Storage Devices in Computer Systems

Main Memory – Cons

Main memory, cache, and registers all work using electrical pulses.
As in flip-flops, before any pulses **there is no state**

- Without electricity, a flip-flop loses it's state.
- Shutting down a computer resets the state of memory, cache, and registers!



Storage Devices in Computer Systems

Mass Storage

- Magnetic Systems
  - Disk
  - Tape
- Optical Systems
  - CD
  - DVD

Storage Devices in Computer Systems

Mass Storage

Pros:

- Can hold huge amounts of data
- Do not lose data when the power is off

Cons:

- Very slow
- Some mass storage devices are *sequential*.
  - You cannot directly access the data you need. For example, tape drives.

Storage Devices in Computer Systems

Mass Storage vs. Main Memory

	Speed	Size	Volatility	Access Method
Mass Storage	Slow	Huge	Very Low	Sequential *or* Random
Main Memory	Fast	Small	High	Random

Mass Storage

On-line vs. Off-line

Definition

On-line means the device or information is connected and readily available to the machine without human intervention.

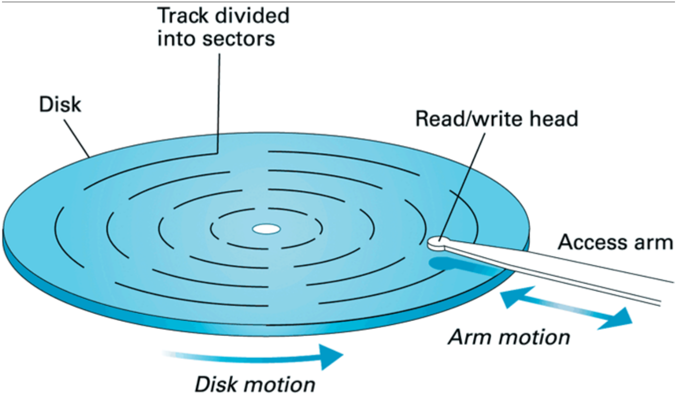
Definition

Off-line means that human intervention is required before the device or information can be accessed by the machine.

Mass Storage

Magnetic Disk Drives

A good example of a magnetic disk drive is the floppy disk drive:



Mass Storage

Hard Disk Drives – Time

$$\text{Disk Speed} = \begin{cases} \text{Access Time} = \begin{cases} \text{Seek Time} \\ \text{Latency Time} \end{cases} \\ \\ \text{Transfer Time} \end{cases}$$

Seek time: The time required to move the read/write heads from one track to another.

Rotation (Latency) time: **Half** the time required for the disk to make a complete rotation.

Transfer time: The rate at which data can be transferred to or from the disk.

Mass Storage

Hard Disk Drives – Time

Example: What is the average access time of a hard disk that spins at 75 revolutions per second[5] with a seek time of 15 milliseconds?

1. Calculate the rotation time: (one half of the rotation time)

$$0.5 \times \frac{1 \text{ second}}{75 \text{ revolutions}} = \frac{1}{150}$$

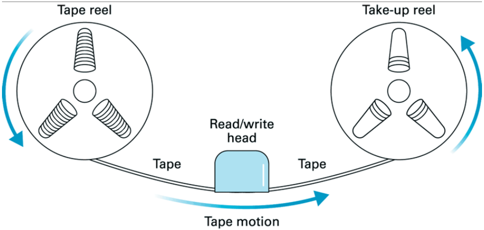
2. Add rotation time and seek time:

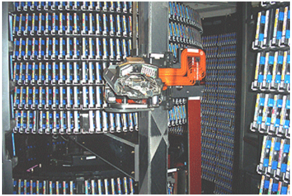
$$0.00667 + 0.015 = 0.02167 \text{ s} = 21.67 \text{ ms}.$$

[5] 75 revolutions per second is the same as 4500 revolutions per minute. 4500 RPM is the slowest hard drive speed for commonly sold disks.

Mass Storage
Tape drive

To access data from a tape drive, the tape is mounted in a device called a tape drive that can read, write, and rewind the tape.

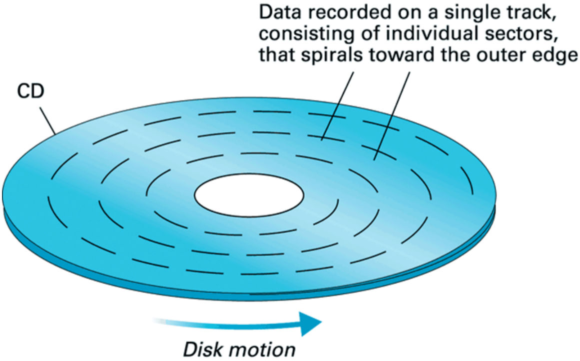




Pros: Low cost, High capacity.
Cons: Extremely slow, so often done overnight.
Magnetic tapes are still used as a popular off-line data storage for archival purposes.

Optical Storage
Compact Disk (CD)

CDs store information on a single track that spirals around the CD, from inside to out.



Questions over Chapters 1.1–1.3?